

# Testing General Relativity on Horizon Scales and the Primordial non-Gaussianity

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The proper general relativistic description of the observed galaxy power spectrum is substantially different from the standard Newtonian description on large scales, providing a unique opportunity to test general relativity on horizon scales. Using the Einstein equations, the general relativistic effects can be classified as two new terms that represent the velocity and the gravitational potential, coupling to the time evolution of galaxy number density and Hubble parameter. Compared to the dominant density and velocity redshift-space distortion terms, the former scales as  $\mathcal{H}/k$  and correlates the real and imaginary parts of the Fourier modes, while the latter scales as  $(\mathcal{H}/k)^2$ , where  $k$  is the comoving wave number and  $\mathcal{H}$  is the conformal Hubble parameter. We use the recently developed methods to reduce the sampling variance and shot noise to show that in an all sky galaxy redshift survey at low redshift the velocity term can be measured at  $10\text{-}\sigma$  confidence level, if one can utilize halos of mass  $M \geq 10^{10} h^{-1} M_\odot$ , while the gravitational potential term itself can only be marginally detected. We also demonstrate that the general relativistic effect is *not* degenerate with the primordial non-Gaussian signature in galaxy bias, and the ability to detect the primordial non-Gaussianity is little compromised.

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The recent discovery of the cosmic acceleration has renewed interest in modifications of gravity on cosmological scales, and tests of general relativity on horizon scales become ever more crucial in determining whether the cosmic acceleration is due to dark energy or the breakdown of general relativity. However, there exists a fundamental limitation to any attempts for testing general relativity on large scales, and decisive conclusions remain elusive due to the cosmic variance limit set by the finite number of measurements available to us.

In the past few decades galaxy clustering has been one of the indispensable tools in cosmology, covering a progressively larger fraction of the sky with increasing redshift depth. With the upcoming dark energy surveys this trend will continue in the future. However, despite the advance in observational frontiers, there remained a few unanswered questions in the theoretical description of galaxy clustering. On horizon scales, the standard Newtonian description naturally breaks down, and a choice of hypersurface of simultaneity becomes an inevitable issue, demanding a fully relativistic treatment of galaxy clustering beyond the current Newtonian description.

In recent work [1, 2], it is shown that a proper relativistic description can be easily obtained by following the observational procedure in constructing the galaxy fluctuation field and its statistics: We need to model observable quantities, rather than theoretically convenient but unobservable quantities, usually adopted in the standard method. While both the relativistic and the standard Newtonian descriptions are virtually indistinguishable in the Newtonian limit, they are substantially different on horizon scales, rendering galaxy clustering measurements a potential probe of general relativity. The relativistic description of galaxy clustering includes two

new terms that scale as velocity and gravitational potential. Compared to the dominant density contribution, they are suppressed by  $\mathcal{H}/k$  and  $(\mathcal{H}/k)^2$  and become important only on large scales, where the comoving wavevector amplitude  $k$  is comparable to the conformal Hubble parameter  $\mathcal{H} = aH$ . Consequently, the identification of these terms just by looking at the galaxy power spectrum is hampered because of sampling variance, and the general relativistic effects unaccounted in the standard Newtonian description may result in systematic errors less than  $1\text{-}\sigma$  for most of the volume available at  $z \leq 3$  in the standard power spectrum analysis [2].

A new multi-tracer method [3] takes advantage of the fact that differently biased galaxies trace the same underlying matter distribution, and it can be used to cancel the randomness of the matter distribution in a single realization of the Universe, eliminating the sampling variance limitation. This method has been used in [4] to investigate the velocity effects of [1, 2], noting that for any given Fourier mode the imaginary part of velocity couples to the real part of density and vice versa. Even with this novel technique, the expected detection level is low [4].

If sampling variance is eliminated, the dominant remaining source of error is shot noise, caused by the discrete nature of galaxies. Recently, a shot noise cancelling technique has been proposed [5] and investigated for detecting primordial non-Gaussianity [6] in combination with the sampling variance cancelling technique. The basis of the method is that by using halo mass dependent weights one can approximate a halo field as the dark matter field and reduce the stochasticity between them. While this works best when comparing halos to dark matter, some shot noise cancelling can also be achieved by comparing halos to each other [6, 7]. This opens up a new opportunity to probe horizon scales and extract cosmological information with higher signal-to-noise ratio. In this *Letter* we explore the possibility of using galaxy power

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spectrum measurements combined with the multi-tracer and shot noise cancelling techniques to test general relativity on cosmological horizon scales. In addition, we comment on the impact of the general relativistic effects in detecting the primordial non-Gaussian signature in galaxy bias.

*GR description.*— A full general relativistic description of galaxy clustering is developed in [1, 2] (see also [8, 9]). Here we make one modification in the adopted linear bias ansatz. Previously, we have adopted the simplest linear bias ansatz, in which the galaxy number density is just a function of the matter density  $n_g = F[\rho_m]$ , both at the same spacetime [10]. However, this ansatz turns out to be rather restrictive, since the time evolution of the galaxy sample is entirely driven by the evolution of the matter density  $\propto (1+z)^3$ . Now we relax this assumption and provide more freedom, while keeping the locality: We allow the galaxy number density at the same matter density to differ depending on its local history, i.e.,  $n_g = F[\rho_m, t_p]$  with  $t_p$  being the proper time measured in the galaxy rest frame. Therefore, when expressed at the observed redshift  $z$ , the galaxy number density can be written as

$$\begin{aligned} n_g &= \bar{n}_g(z) [1 + b(\delta_m - 3\delta z) - b_t \delta z_v] \\ &= \bar{n}_g(z) \left[ 1 + b \delta_m^{(v)} - e \delta z_v \right], \end{aligned} \quad (1)$$

where the lapse  $\delta z$  in the observed redshift  $z$  is defined as  $1+z = (1+\delta z)/a$  and the expansion factor is  $a$ . The script  $v$  indicates quantities are evaluated in the dark matter comoving gauge ( $v = 0$ ), and this gauge condition coincides with the synchronous gauge, as pressureless dark matter can freely fall in the rest frame (see, e.g., [12]).

The galaxy bias factor is  $b = \partial \ln \bar{n}_g / \partial \ln \rho_m|_{t_p}$  and the time evolution of the galaxy number density due to its local history is  $b_t = \partial \ln \bar{n}_g / \partial \ln(1+z)|_{\rho_m}$ . Therefore, the total time evolution of the mean galaxy number density is proportional to the evolution bias [13],  $e = d \ln \bar{n}_g / d \ln(1+z) =$

$3b + b_t$ . This biasing scheme in Eq. (1) is consistent with [8, 9, 14, 15], and our previous bias ansatz corresponds to  $b_t = 0$ . Physically, the presence of long wavelength modes affects the local dynamics of galaxy formation by changing the local curvature and thus the expansion rate, and these are modulated by the Laplacian of the comoving curvature [14].

Therefore, with this more physically motivated bias ansatz, the general relativistic description of the observed galaxy fluctuation is [1, 2]

$$\begin{aligned} \delta_{\text{GR}} &= \left[ b \delta_m^{(v)} - e \delta z_v \right] + \alpha_\chi + 2 \varphi_\chi + V + 3 \delta z_\chi \\ &+ 2 \frac{\delta \mathcal{R}}{r} - H \frac{d}{dz} \left( \frac{\delta z_\chi}{\mathcal{H}} \right) - 5p \delta \mathcal{D}_L - 2 \mathcal{K}, \end{aligned} \quad (2)$$

where the luminosity function slope of the source galaxy population is  $p$ , the comoving line-of-sight distance is  $r$ , the fluctuation in the luminosity distance is  $\delta \mathcal{D}_L$ , the temporal and spatial metric perturbations are  $\alpha_\chi$  and  $\varphi_\chi$ , the line-of-sight velocity is  $V$ , and the gauge-invariant radial displacement and lensing convergence are  $\delta \mathcal{R}$  and  $\mathcal{K}$ . We have ignored the vector and tensor contributions to  $\delta_{\text{GR}}$ . The subscript  $\chi$  indicates quantities are evaluated in the conformal Newtonian gauge ( $\chi = 0$ ). We emphasize that compared to [1, 2] it is only the terms in the square bracket that are affected by the choice of linear bias ansatz.

Noting that Eq. (2) can be most conveniently computed in the conformal Newtonian gauge, we first remove the lapse term  $\delta z_v$  by using its gauge transformation property  $\delta z_v = \delta z_\chi + \mathcal{H} v_\chi / k$ , but we keep the comoving gauge matter fluctuation  $\delta_m^{(v)}$  to be consistent with the convention in the literature. In Fourier space, a further simplification can be made by ignoring the projected quantities such as the gravitational lensing and the integrated Sachs-Wolfe contributions, which are important only for the pure transverse modes ( $k^\parallel = 0$ ) [2, 16]. Equation (2) in Fourier space is then

$$\begin{aligned} \delta_{\text{GR}} &= \left[ b \delta_m^{(v)} - \mu_k^2 \frac{k v_\chi}{\mathcal{H}} \right] - e \frac{\mathcal{H}}{k} v_\chi + 2 \varphi_\chi - \frac{\varphi'_\chi}{\mathcal{H}} + i \mu_k \frac{k \alpha_\chi}{\mathcal{H}} - i \mu_k \frac{v'_\chi}{\mathcal{H}} + \left[ e + 1 - \frac{1+z}{H} \frac{dH}{dz} + (5p-2) \left( 1 - \frac{1}{\mathcal{H}r} \right) \right] \alpha_\chi \\ &+ \left[ e - 1 - \frac{1+z}{H} \frac{dH}{dz} + (5p-2) \left( 1 - \frac{1}{\mathcal{H}r} \right) \right] i \mu_k v_\chi = \delta_{\text{Newt}} - i \mu_k \left[ e - \frac{1+z}{H} \frac{dH}{dz} + (5p-2) \left( 1 - \frac{1}{\mathcal{H}r} \right) \right] \frac{f \delta_m^{(v)}}{k/\mathcal{H}} \\ &+ \left\{ e f - \frac{3}{2} \Omega_m(z) \left[ e + f - 2 - \frac{1+z}{H} \frac{dH}{dz} + (5p-2) \left( 1 - \frac{1}{\mathcal{H}r} \right) \right] \right\} \frac{\delta_m^{(v)}}{(k/\mathcal{H})^2} \equiv \delta_{\text{Newt}} - i \mu_k \frac{\mathcal{R} \delta_m^{(v)}}{k/\mathcal{H}} + \frac{\mathcal{P} \delta_m^{(v)}}{(k/\mathcal{H})^2}, \end{aligned} \quad (3)$$

where  $f$  is the logarithmic growth rate and we defined the two dimensionless coefficients  $\mathcal{R}$  and  $\mathcal{P}$  in the last equality. We have used the Einstein equations in the second equality to express  $\alpha_\chi$ ,  $\varphi_\chi$ , and  $v_\chi$  in terms of  $\delta_m^{(v)}$ . The standard Newtonian description of the observed galaxy fluctuation is  $\delta_{\text{Newt}} = b \delta_m^{(v)} - \mu_k^2 k v_\chi / \mathcal{H} = (b + f \mu_k^2) \delta_m^{(v)}$ , which can be contrasted with  $\delta_{\text{GR}}$  in Eq. (2).

Apparent from their spatial dependence in Eq. (3), the coefficients  $\mathcal{R}$  and  $\mathcal{P}$  originate from the velocity and gravitational potential. While  $\mathcal{P}$  is purely relativistic, some contributions to  $\mathcal{R}$  may be considered non-relativistic, since they could be written down in Newtonian dynamics, simply as a coupling of velocity from the Doppler effect with the time evolution of the galaxy number density. However, correct estimates of

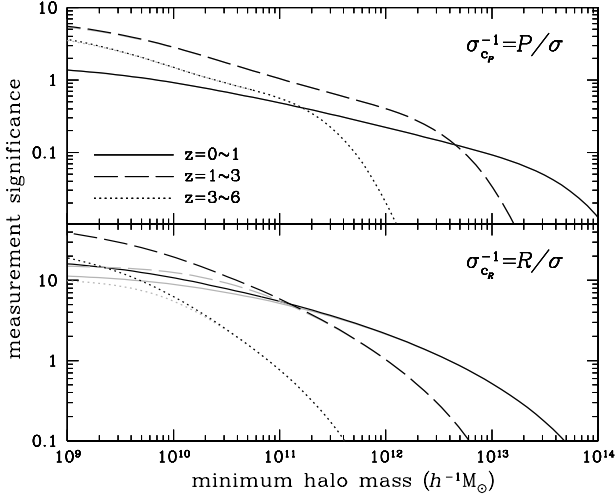


FIG. 1: Predicted measurement significance of general relativistic effects in the galaxy power spectrum. All the halos above a given minimum mass are utilized to take advantage of the multi-tracer method [3, 6]. Three curves represent different survey redshift ranges with corresponding volume  $V = 59, 410, 600 (h^{-1}\text{Gpc})^3$ . Gray curves show the degradation of the measurement significance, when the evolution bias  $e$  and the luminosity function slope  $p$  are marginalized over. For the volume-limited sample with constant comoving number density,  $(\mathcal{P}, \mathcal{R}) = (1.7, 1.6)$ ,  $(2.1, 1.5)$ , and  $(2.2, 1.5)$  at each redshift.

the  $\mathcal{R}$  value (and  $\mathcal{P}$ ) require fully relativistic treatments, as perturbations like velocity and potential are gauge dependent quantities. Furthermore, while Eq. (2) can be derived with the minimal assumption that the spacetime is described by a perturbed FLRW metric and photons follow geodesics, the coefficients  $\mathcal{R}$  and  $\mathcal{P}$  in Eq. (3) are obtained by utilizing the Einstein equations. Therefore, measuring  $\mathcal{R}$  and  $\mathcal{P}$  is equivalent to a direct measurement of the relativistic contributions, *not* through their influence on the matter density fluctuation, and we collectively refer to the coefficients  $\mathcal{R}$  and  $\mathcal{P}$  as the general relativistic effects in the galaxy power spectrum.

*Multi-tracer shot noise cancelling method.*— We consider multiple galaxy samples with different bias factors for measuring the general relativistic effects in the galaxy power spectrum. Using a vector notation, the observed galaxy fluctuation fields can be written as

$$\delta_{\text{GR}}^{\text{obs}} = \left[ \mathbf{b}_0 + f\mu_k^2 \mathbf{I} + \frac{c_P \mathcal{P}}{(k/\mathcal{H})^2} - i\mu_k \frac{c_R \mathcal{R}}{k/\mathcal{H}} \right] \delta_m^{(v)} + \varepsilon, \quad (4)$$

where  $\mathbf{b}_0$ ,  $\mathbf{I}$ ,  $\varepsilon$  are the linear bias, the multi-dimensional identity, and the residual-noise field vectors. Similarly, the coefficients  $\mathcal{R}$  and  $\mathcal{P}$  become vectors with different values of  $e$  and  $p$  for each sample. We introduced two new parameters  $c_R$  and  $c_P$  to generalize the measurement significance of the coefficients  $\mathcal{R}$  and  $\mathcal{P}$  to the case of multiple galaxy samples — they are  $c_R = c_P = 1$  in general relativity, and measurements of these two parameters amount to the measurement significance of the two vectors  $\mathcal{R}$  and  $\mathcal{P}$ . By definition the noise-field is independent of the matter fluctuation  $\langle \varepsilon \delta_m^{(v)} \rangle = 0$ , and the square bracket in Eq. (4) defines the effective bias vector  $\mathbf{b}$ .

In order to assess our ability to measure the general relativistic effects in the galaxy power spectrum, we employ the Fisher information matrix:

$$F_{ij}(k, \mu_k) = \frac{\alpha}{1 + \alpha} \text{Re}(\gamma_{ij}) + \frac{\text{Re}(\beta_i \beta_j - \alpha \beta_i \beta_j^*)}{(1 + \alpha)^2}, \quad (5)$$

where  $\alpha = \mathbf{b}^\dagger \mathcal{E}^{-1} \mathbf{b} P_m$ ,  $\beta_i = \mathbf{b}^\dagger \mathcal{E}^{-1} \mathbf{b}_i P_m$ ,  $\gamma_{ij} = \mathbf{b}_i^\dagger \mathcal{E}^{-1} \mathbf{b}_j P_m$ , the shot noise matrix  $\mathcal{E} = \langle \varepsilon \varepsilon^\dagger \rangle$ , the matter power spectrum in the comoving gauge is  $P_m(k)$ , the parameters  $\theta_i = c_R, c_P, e, p$ , and  $\mathbf{b}_i$  is the derivative of  $\mathbf{b}$  with respect to the parameters  $\theta_i$ . This formula is a straightforward extension of the Fisher matrix derived in [6], with the effective bias vector  $\mathbf{b}$  being a complex vector. The imaginary part arises solely from the  $\mathcal{R}$ -term in Eq. (4) and its derivative.

*Measurement significance.*— For definiteness, we consider full sky surveys with three different redshift ranges and adopt a set of cosmological parameters consistent with the WMAP7 results [17]. Given the survey volume  $V$ , the Fisher matrix in Eq. (5) is summed over the Fourier volume, where  $k_{\min} = 2\pi/V^{1/3}$  and  $k_{\max} = 0.03 h \text{Mpc}^{-1}$ . To model the Fisher matrix parameters  $\alpha, \beta_i, \gamma_{ij}$ , we adopt the halo model description in [6, 7]; It has been well tested against a suite of  $N$ -body simulations with Gaussian and non-Gaussian initial conditions. We assume that the galaxy samples have a constant comoving number density ( $e = 3\mathbf{I}$ ) and they are selected in a volume-limited survey to eliminate the magnification bias (or  $p = 0.4\mathbf{I}$  in practice), but we marginalize over  $e$  and  $p$  in constraining  $c_R$  and  $c_P$  with priors  $\sigma_e = 0.1$  and  $\sigma_p = 0.05$  [18].

Figure 1 shows the predicted measurement significance of the general relativistic effects. In our most optimistic scenario, the velocity term  $c_R$  can be measured at more than  $10\text{-}\sigma$  confidence level at  $z \leq 1$ , while the gravitational potential  $c_P$  is at  $1\text{-}\sigma$  significance. At higher redshift, though the increase in the survey volume is partially cancelled by the lower abundance of halos at a fixed mass, a substantial improvement can be achieved by going beyond  $z = 1$ . However, as the gray curves show, the uncertainties in  $e$  and  $p$  need to be controlled beyond the current observational limit, before further improvement can be realized.

Compared to the estimates  $S/N \lesssim 1$  obtained by using the standard power spectrum analysis [2], the multi-tracer method dramatically enhances the measurement significance. Furthermore, a method of measuring the imaginary part in the galaxy power spectrum of two tracers [4] is fully implemented in our complex covariance matrix as off-diagonal terms. The result in [4] would correspond to  $\mathcal{R}/\sigma \simeq 1.8$  at  $z \leq 1$ . Finally, any degeneracy with cosmological parameters in measuring the relativistic effects is largely eliminated, since the underlying matter distribution is cancelled [3].

*Primordial non-Gaussianity.*— We extend our formalism to the primordial non-Gaussian signature in galaxy bias [19] and introduce additional parameter  $f_{\text{NL}}$ . Here we only consider the simplest local form of primordial non-Gaussianity to demonstrate how it can be implemented in the general relativistic description, and ignore scale-independent and scale-dependent corrections (see, e.g., [20]).

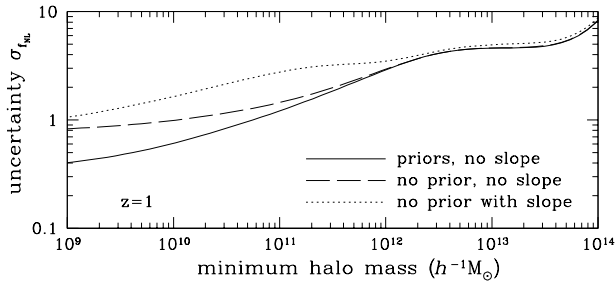


FIG. 2: Predicted constraints on the primordial non-Gaussianity  $f_{\text{NL}}$  from galaxy power spectrum measurements. To facilitate the comparison, the constraints on  $f_{\text{NL}}$  are obtained by using the same survey specifications as in [6]:  $V \simeq 50 (h^{-1} \text{Gpc})^3$  centered at  $z = 1$ . Various curves show  $\sigma_{f_{\text{NL}}}$  with different priors on  $e$  and  $p$ .

The primordial non-Gaussian signature in galaxy bias can be readily implemented in our full general relativistic description with the Gaussian bias factor in Eq. (2) replaced by

$$b \rightarrow b + 3f_{\text{NL}}(b - 1)\delta_c \frac{\Omega_m(z)\mathcal{H}^2}{T_\varphi(k, z)k^2}, \quad (6)$$

or equivalently by adding additional term  $3f_{\text{NL}}(b - 1)\delta_c\Omega_m(z)/T_\varphi(k, z)$  to the coefficient  $\mathcal{P}$  in Eq. (3), where  $T_\varphi$  is the transfer function of the curvature perturbation (see also [9, 14, 15]).

In obtaining the constraint  $\sigma_{f_{\text{NL}}}$  on primordial non-Gaussianity, we set  $c_{\mathcal{R}} = c_{\mathcal{P}} = 1$  and marginalize over  $e$  and  $p$ . With the current uncertainties in  $e$  and  $p$ , the constraint  $\sigma_{f_{\text{NL}}}$  (solid in Fig. 2) is nearly identical to the unmarginalized constraint. The dashed curve shows that even with no priors on  $e$  and  $p$ ,  $\sigma_{f_{\text{NL}}}$  is not inflated except in the regime with  $\sigma_{f_{\text{NL}}} \lesssim 2$ , because  $\mathcal{R}$  and  $\mathcal{P}$  are affected simultaneously by  $e$  and  $p$  but only  $\mathcal{P}$  by  $f_{\text{NL}}$ . Furthermore, the unique dependence of  $f_{\text{NL}}$  on  $b - 1$  and  $T_\varphi$  in Eq. (6) provides the multi-tracer method with more leverage to separate it from the general relativistic effect.

Finally, we allow  $e$  and  $p$  to vary as a function of mass with two logarithmic slope parameters  $\alpha_e$  and  $\alpha_p$ :  $e =$

$e_0 \mathbf{I} + \alpha_e \ln(\mathbf{M}/M_0)$ ,  $p = p_0 \mathbf{I} + \alpha_p \ln(\mathbf{M}/M_0)$  with  $e_0 = 3$ ,  $p_0 = 0.4$ ,  $\alpha_e = \alpha_p = 0$ , and  $M_0 = 10^{12} h^{-1} M_\odot$ . The effects of  $\alpha_e$  and  $\alpha_p$  (dotted in Fig. 2) are sufficiently different from that of  $f_{\text{NL}}$ , and  $\sigma_{f_{\text{NL}}}$  asymptotically reaches the floor set by the uncertainties in  $e$  and  $p$ . This demonstrates that the general relativistic effects in the galaxy power spectrum are *not* degenerate with the primordial non-Gaussian signature. However, if  $f_{\text{NL}}$  were to be constrained below unity, similar precision needs to be achieved in predicting  $\mathcal{R}$  and  $\mathcal{P}$ . This requirement can be relaxed by using independent mass estimates, as  $p$  drops out in Eq. (3), further separating  $e$  from  $f_{\text{NL}}$ .

*Discussion.*— We have demonstrated that in a typical galaxy survey at low redshift general relativistic effects in the galaxy power spectrum can be measured at very high significance by using the multi-tracer method, providing a unique opportunity to test general relativity on horizon scales. We also show how the primordial non-Gaussian effect in galaxy bias can be implemented in the full general relativistic description, and quantitatively proved that the ability to detect primordial non-Gaussianity is *little* compromised by the presence of general relativistic effects.

Proper application of our method to observations will require a few key investigations beyond our simple treatment considered in this *Letter*. First, while we treated multiple galaxy samples as halos in multiple mass bins, it is shown [6] that a large scatter in the mass-observable relation, i.e.,  $\sigma_{\ln M} = 0.5$ , would degrade the signal-to-noise ratio by less than a factor of two. Second, our prediction is based on the halo model description of the shot noise matrix, which is tested only for halos at  $M \geq 10^{12} h^{-1} M_\odot$ , and our prediction at  $M \leq 10^{12} h^{-1} M_\odot$  is an extrapolation.

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